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Physicist and electronic engineer

**Is Mathematics Invented or Discovered?**

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Mathematics is the language of science and has enabled mankind to make extraordinary technological advances. There is no question that the logic and order that underpins mathematics, has served us in describing the patterns and structure we find in nature.

The successes that have been achieved, from the mathematics of the cosmos down to electronic devices at the microscale, are significant. Einstein remarked, "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?"

Amongst mathematicians and scientists there is no consensus on this fascinating question. The various types of responses to Einstein's conundrum include:

**1) Math is innate.**The reason mathematics is the natural language of science, is that the universe is underpinned by the same order. The structures of mathematics are intrinsic to nature. Moreover, if the universe disappeared tomorrow, our eternal mathematical truths would still exist. It is up to us to discover mathematics and its workings--this will then assist us in building models that will give us predictive power and understanding of the physical phenomena we seek to control. This rather romantic position is what I loosely call mathematical Platonism.

**2) Math is a human construct.**The only reason mathematics is admirably suited describing the physical world is that we invented it to do just that. It is a product of the human mind and we make mathematics up as we go along to suit our purposes. If the universe disappeared, there would be no mathematics in the same way that there would be no football, tennis, chess or any other set of rules with relational structures that we contrived. Mathematics is not discovered, it is invented. This is the non-Platonist position.

**3) Math is not so successful.**Those that marvel at the ubiquity of mathematical applications have perhaps been seduced by an overstatement of their successes. Analytical mathematical equations only approximately describe the real world, and even then only describe a limited subset of all the phenomena around us. We tend to focus on those physical problems for which we find a way to apply mathematics, so overemphasis on these successes is a form of "cherry picking." This is the realist position.

**4) Keep calm and carry on.** What matters is that mathematics produces results. Save the hot air for philosophers. This is called the "shut up and calculate" position.

The debate over the fundamental nature of mathematics is by no means new, and has raged since the time of the Pythagoreans. Can we use our hindsight now to shed any light on the above four positions?

A recent development within the last century was the discovery of fractals. Beautiful complex patterns, such as the Mandelbrot set, can be generated from simple iterative equations. Mathematical Platonists eagerly point out that elegant fractal patterns are common in nature, and that mathematicians clearly discover rather than invent them. A counterargument is that any set of rules has emergent properties. For example, the rules of chess are clearly a human contrivance, yet they result in a set of elegant and sometimes surprising characteristics. There are infinite numbers of possible iterative equations one can possibly construct, and if we focus on the small subset that result in beautiful fractal patterns we have merely seduced ourselves.

Take the example of infinite monkeys on keyboards. It appears miraculous when an individual monkey types a Shakespeare sonnet. But when we see the whole context, we realize all the monkeys are merely typing gibberish. In a similar way, it is easy to be seduced into thinking that mathematics is miraculously innate if we are overly focused on its successes, without viewing the complete picture.

The non-Platonist view is that, first, all mathematical models are approximations of reality. Second, our models fail, they go through a process of revision, and we invent new mathematics as needed. Analytical mathematical expressions are a product of the human mind, tailored for the mind. Because of our limited brainpower we seek out compact elegant mathematical descriptions to make predictions. Those predictions are not guaranteed to be correct, and experimental verification is always required. What we have witnessed over the past few decades, as transistor sizes have shrunk, is that nice compact mathematical expressions for ultra small transistors are not possible. We could use highly cumbersome equations, but that isn't the point of mathematics. So we resort to computer simulations using empirical models. And this is how much of cutting edge engineering is done these days.

The realist picture is simply an extension of this non-Platonist position, emphasizing that compact analytical mathematical expressions of the physical world around us are not as successful or ubiquitous as we'd like to believe. The picture that consistently emerges is that all mathematical models of the physical world break down at some point. Moreover, the types of problems addressed by elegant mathematical expressions are a rapidly shrinking subset of all the currently emerging scientific questions.

But why does this all matter? The "shut up and calculate" position tells us to not worry about such questions. Our calculations come out the same, no matter what we personally believe; so keep calm and carry on.

I, for one, believe the question is important. My personal story is that I used to be a Platonist. I thought all mathematical forms were reifiedand waiting to be discovered. This meant that I philosophically struggled with taking limits to infinity, for example. I merely got used to it and accepted it under sufferance. During my undergraduate days, I had a moment of enlightenment and converted to non-Platonism. I felt a great burden lift from my shoulders. Whilst this never affected my specific calculations, I believe a non-Platonist position gives us greater freedom of thought. If we accept that mathematics is invented, rather than discovered, we can be more daring, ask deeper questions, and be motivated to create further change.

Remember how irrational numbers petrified the bejesus out of the Pythagoreans? Or the interminable time it took mankind to introduce a zero into arithmetic? Recall the centuries of debate that occurred over whether negative numbers are valid or not? Imagine where science and engineering would be today if this argument was resolved centuries earlier. It is the ravages of Platonist-like thinking that have held back progress. I argue that a non-Platonist position frees us from an intellectual straightjacket and accelerates progress.

**More information:**Derek Abbott, "The reasonable ineffectiveness of mathematics,"[*Proceedings of the IEEE*](http://dx.doi.org/10.1109/JPROC.2013.2274907), Vol. 101, No. 10, pp. 2147-2153 , 2013.

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## APR 1, 2010

## [Is Mathematics Invented or Discovered?](http://www.scienceandreligiontoday.com/2010/04/01/is-mathematics-invented-or-discovered/)

**From**[**Robert Lawrence Kuhn**](http://www.closertotruth.com/robert-lawrence-kuhn)**, host and creator of**[**Closer To Truth**](http://www.closertotruth.com/)**:**

Mathematics describes the real world of atoms and acorns, stars and stairs. Simple abstract equations define complex physical things beautifully, elegantly. Why should this be? The more I think about it, the more astonished I get.
Albert Einstein said: “The most incomprehensible thing about the universe is that it is comprehensible.” Physicist Eugene Wigner wrote of “the unreasonable effectiveness of mathematics” in science. So is mathematics invented by humans, like chisels and hammers, cars and computers, music and art? Or is mathematics discovered, always out there, somewhere, like mysterious islands waiting to be found? The question probes the deepest secrets of existence.
[**Roger Penrose**](http://www.closertotruth.com/participant/Roger-Penrose/75), one of the world’s most distinguished mathematicians, says that “people often find it puzzling that something abstract like mathematics could really describe reality.” But you cannot understand atomic particles and structures, such as gluons and electrons, he says, except with mathematics. “These equations are fantastically accurate,” he remarks. “Newton’s theory has a precision of something like one part in 10 million. Einstein’s theory something like one part in 100 billion.”
Penrose famously believes that mathematics has an independent existence, a “Platonic existence” (following Plato’s “forms”) that is radically distinct from physical space and time. “Certainly,” he says, “mathematicians view mathematics as something out there, which seems to have a reality independent of the ordinary kind of reality of things like chairs, which we normally think of as real. It’s sometimes referred to as a ‘Platonic world,’ a Platonic reality. … I like to think of mathematics as a bit like geology or archeology, where you’re really exploring beautiful things out there in the world, which have been out there, in fact, for ages and ages and ages, and you’re revealing them for the first time.”
Penrose describes three kinds of “worlds”— physical, mental, and mathematical (Platonic)—and he wonders whether “the mathematical reality of the Platonic world gives reality to these worlds.”
But not everyone agrees. Philosopher [**Mark Belaguer**](http://www.closertotruth.com/participant/Mark-Balaguer/8) notes that “there are tons and tons of mathematical structures that are of no use at all in studying the physical world” and the reason that “mathematicians started studying those structures that turned out to be useful was because they lived in the physical world.”
But do abstract mathematical objects really exist in the world? Belaguer cites four opposing views: the “mentalistic” view that mathematical objects are all in our head; the “physicalistic” view that mathematical objects exist in the physical world; the Platonic view that the mathematical objects are nonphysical and nonmental abstract objects; and the “anti-realist” view that there aren’t mathematical objects at all.
(Belaguer defines an “abstract object,” such as a number, as “not physical, not mental, and not entering into causal relations. So an abstract object is not like any object we ever encounter in our ordinary lives. And belief in those kinds of abstract objects is called Platonism because it was famously Plato’s view that there were such things.”)
Belaguer asserts: “The right kind of Platonism is the strongest kind of realism you can have. And ‘fictionalism’ is the strongest kind of anti-realism you can have because the mathematics is literally false.” Interestingly, he defends both of these diametrically opposed views because, he says, “the only thing they disagree on is: Do the abstract objects exist or not?” And, he concludes, “it doesn’t look like we have any way of knowing whether they exist. So we can’t discover whether Platonism or fictionalism is the right view.”
But is one or the other correct? No, Belaguer says, “there’s no fact of the matter about whether the abstract objects exist. … There’s no right answer to it.”
The effectiveness of mathematics is clear. Why, then, is the essence of mathematics so foggy? Is math mental, physical, Platonic, or just not real?
I ask mathematician [**Gregory Chaitin**](http://www.closertotruth.com/participant/Gregory-Chaitin/17). “When you’re a mathematician and you find something that feels really fundamental,” he says, “you may think that if you hadn’t found it, somebody else would have because in some sense it’s got to be there. But some mathematics feels much more contrived. … If you look into the inner recesses of many mathematicians, and I include myself, you find that we have this theological medieval belief in this Platonic world of perfect ideas of mathematical concepts.” But, he muses, “is it all a game that we just invent as we go along?”
Chaitin offers his “final conclusion after a lifetime obsessed with mathematics.” Mathematicians, he says, “should behave a little bit more like experimental scientists do.” He argues that “if they do computer experiments and see that something seems to be the case, but they can’t prove it, and yet this something is a very useful truth if it were true, then maybe they should add that as a new axiom. Mathematicians will reel back in horror, but I think my work pushes in this direction. I’ve been forced against my will toward saying that mathematics is empirical—or, to put it in other words, we invent it as we go.” Concluding, Chaitin laments, “I’m not quite sure where we are.”
Invented or discovered? Chaitin’s ideas about math have changed, from certainty to uncertainty about whether math has always existed. If, after a lifetime in math, Chaitin is “not quite sure,” where does that leave me? And, yes, it does matter. Math is fundamental to existence!
In a new way of thinking, physicist [**Stephen Wolfram**](http://www.closertotruth.com/participant/Stephen-Wolfram/123) offers the shocking idea that simple rules, not complex mathematics, construct reality. “For a long time, I’ve been interested in the essence of mathematics,” Wolfram says. “Is today’s mathematics the only possible mathematics, or is it a mathematics that is sort of a great artifact of our civilization?” His “resounding” conclusion is that “the mathematics that we have today is in fact a historical artifact. Now throughout history, that’s not what mathematicians have tended to conclude. They’ve [generally] thought that mathematics is the most general possible formal abstract system.”
He continues: “If you look at the history of mathematics, that’s certainly not how it originally started out. In ancient Babylon, arithmetic was for commerce, geometry for land surveying.” Then came “the progressive generalization of arithmetic and geometry, plus one key methodological idea: that one can make theorems and abstract proofs of those theorems,” he says. “One can ask the question: If one arbitrarily looks at formal systems, will they tend to have the character of mathematics as we know it today? Will they tend to have the feature that one can successfully prove theorems? I think in both cases the answer is no, not really.”
Wolfram suggests that we “deconstruct mathematics” by recognizing that all our mathematics are based on a certain set of axioms, which are quite simple. “But there’s a whole universe of possible mathematics out there,” he states. “What are they like? And where does our particular mathematics lie in this universe of possible mathematics? Is it possible mathematics number one? Is it possible mathematics number 10? Is it possible mathematics number quintillion? … The answer depends on exactly how you enumerate the space. But roughly, our mathematics is about the 50,000th possible axiom system. So right there in the universe of possible axiom systems, the universe of possible mathematics, there’s logic.”
He goes on: “I suspect that if we were to just sort of ask mathematical questions arbitrarily, the vast majority of them would end up turning out to be unsolvable. We just don’t see it because the particular way that our mathematics has progressed historically has tended to avoid it. Now, you might say, ‘But mathematics is a good model of the natural world.’ I think there’s kind of a circular argument here because what’s happened is that those things which have been successfully addressed in science when studying the natural world are just those things that mathematical methods have successfully allowed us to address.”
Wolfram appreciates that human mathematics is “one of the wonderful things that has been produced by a huge amount of human effort.” Nonetheless, he concludes, it is an artifact.
Thus, Wolfram rejects the idea that our mathematics has deep significance. Rather, he looks to the vastly large “space of all possible mathematics.” But math as mere artifact still troubles me.
Nobel laureate [**Frank Wilczek**](http://www.closertotruth.com/participant/Frank-Wilczek/121) tells me that mathematics is both invented and discovered,” but he thinks “it’s mostly discovered.” Mathematics, he says, “is the process of taking axioms, definite sets of assumptions, and drawing out the consequences. So, devising axioms is invention, and drawing out the consequences is discovery.”
He explains that, “Occasionally, you have to introduce new sets of axioms like the passage from Euclidean geometry to non-Euclidean geometry. These are epical events in mathematics, which, in a sense, are inventions.”
But isn’t the world constructed with non-Euclidean geometry, such as Einstein’s theory of relativity, so that it was somehow always there?
“Inventions have to come from somewhere,” Wilczek responds. “So they could be inspired by natural phenomena. … You can invent [all kinds of] axioms, but most of them won’t be interesting. And the ones that are interesting are discoveries, so even the inventions have some element of discovery. So as I said, mathematics is more discovered than invented, and this only makes it more so.”
So, is mathematics invented or discovered? Here’s what we know. Mathematics describes the physical world with remarkable precision. Why? There are two possibilities.
First, math somehow underlies the physical world, generates it. Or second, math is a human description of how we describe certain regularities in nature, and because there is so much possible mathematics, some equations are bound to fit.
As for the essence of mathematics, there are four possibilities. Only one is really true. Math could be: physical, in the real world, actually existing; mental, in the mind, only a human construct; Platonic, nonphysical, nonmental abstract objects; or fictional, anti-realist, utterly made up. Math is physical or mental or Platonic or fictional. Choose only one.
In peering down the dark well of deep reality, mathematics brings us closer to truth.

<http://www.scienceandreligiontoday.com/2010/04/01/is-mathematics-invented-or-discovered/>

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# Math: Discovered, Invented, or Both?

### By [Mario Livio](http://www.pbs.org/wgbh/nova/blogs/physics/author/mario-livio/) on Mon, 13 Apr 2015

 “The miracle of the appropriateness of the language of mathematics to the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.”

Eugene Wigner wrote these words in his 1960 article “[The Unreasonable Effectiveness of Mathematics in the Natural Sciences](http://math.northwestern.edu/~theojf/FreshmanSeminar2014/Wigner1960.pdf).” The Nobel prize-winning physicist’s report still captures the uncanny ability of mathematics not only to describe and explain, but to predict phenomena in the physical world.



Credit: Flickr user [Barney Livingston](https://www.flickr.com/photos/barnoid/), adapted under a [Creative Commons license](https://creativecommons.org/licenses/by-nc-sa/2.0/).

How is it possible that all the phenomena observed in classical electricity and magnetism can be explained by means of just four mathematical equations? Moreover, physicist James Clerk Maxwell (after whom those four equations of electromagnetism are named) showed in 1864 that the equations predicted that varying electric or magnetic fields should generate certain propagating waves. These waves—the familiar electromagnetic waves (which include light, radio waves, x-rays, etc.)—were eventually detected by the German physicist Heinrich Hertz in a series of experiments conducted in the late 1880s.

And if that is not enough, the modern mathematical theory which describes how light and matter interact, known as quantum electrodynamics (QED), is even more astonishing. In 2010 a group of physicists at Harvard University determined the magnetic moment of the electron (which measures how strongly the electron interacts with a magnetic field) to a precision of less than one part in a trillion. Calculations of the electron’s magnetic moment based on QED reached about the same precision and the two results agree! What is it that gives mathematics such incredible power?

The puzzle of the power of mathematics is in fact even more complex than the above examples from electromagnetism might suggest. There are actually two facets to the “unreasonable effectiveness,” one that I call active and another that I dub passive. The active facet refers to the fact that when scientists attempt to light their way through the labyrinth of natural phenomena, they use mathematics as their torch. In other words, at least some of the laws of nature are formulated in directly applicable mathematical terms. The mathematical entities, relations, and equations used in those laws were developed for a specific application. Newton, for instance, formulated the branch of mathematics known as calculus because he needed this tool for capturing motion and change, breaking them up into tiny frame-by-frame sequences. Similarly, string theorists today often develop the mathematical machinery they need.

Passive effectiveness, on the other hand, refers to cases in which mathematicians developed abstract branches of mathematics with absolutely no applications in mind; yet decades, or sometimes centuries later, physicists discovered that those theories provided necessary mathematical underpinnings for physical phenomena. Examples of passive effectiveness abound. Mathematician Bernhard Riemann, for example, discussed in the 1850s new types of geometries that you would encounter on surfaces curved like a sphere or a saddle (instead of the flat plane geometry that we learn in school). Then, when Einstein formulated his theory of General Relativity (in 1915), Riemann’s geometries turned out to be precisely the tool he needed!

At the core of this math mystery lies another argument that mathematicians, philosophers, and, most recently, cognitive scientists have had for a long time: Is math an invention of the human brain? Or does math exist in some abstract world, with humans merely discovering its truths? The debate about this question continues to rage today.

Personally, I believe that by asking simply whether mathematics is discovered or invented, we forget the possibility that mathematics is an intricate combination of inventions and discoveries. Indeed, I posit that humans invent the mathematical concepts—numbers, shapes, sets, lines, and so on—by abstracting them from the world around them. They then go on to discover the complex connections among the concepts that they had invented; these are the so-called theorems of mathematics.

I must admit that I do not know the full, compelling answer to the question of what is it that gives mathematics its stupendous powers. That remains a mystery.

**Go Deeper**
Editor’s picks for further reading

NOVA: [The Great Math Mystery](http://www.pbs.org/wgbh/nova/physics/great-math-mystery.html)
Is math invented by humans, or is it the language of the universe? NOVA takes on this question in a new film premiering April 15, 2015 at 9pm on most PBS stations.

NOVA: [Describing Nature with Math](http://www.pbs.org/wgbh/nova/physics/describing-nature-math.html)
How do scientists use mathematics to define reality? And why? Peter Tyson investigates two millennia of mathematical discovery.

The Washington Post: [The Structure of Everything](http://www.washingtonpost.com/wp-dyn/content/article/2009/02/05/AR2009020502876.html)
Learn more about the “unreasonable effectiveness of mathematics” in this review of Mario Livio’s book “Is God a Mathematician?”

**Creation v Discovery**

Conversations on Mind, Matter and Mathematics

September 29 1995

Does mathematics have an existence independent of our physical world? Do mathematicians discover theorems, rather than invent them? Such questions have exercised the minds of philosophers and mathematicians since the time of Plato, and many books have addressed the issues.

What is unusual and topical about this book is that it records a dialogue between a mathematician, Alain Connes, and a biologist, Jean-Pierre Changeux. Each is an international authority in his field and both are professors at the Coll ge de France, Paris, so that they speak from first-hand experience and the intellectual level is high. As a neurophysiologist Changeux is particularly interested in the structure and organisation of the brain. He is intrigued by the capacity of the brain to handle the kind of sophisticated intellectual abstractions represented by mathematics, and his dialogue with Connes is an attempt to get to grips with this problem.

Most mathematicians would agree that their subject has evolved from the traditional disciplines of arithmetic and geometry. We have built an elaborate superstructure on these twin foundations and so the nature of "mathematical reality" can be adequately dealt with by posing basic questions about the integers 1,2,3... and about the Euclidean geometry of triangles, circles, and so on. Connes is an unabashed Platonist. For him there is an "archaic mathematical reality" represented by objects such as circles or integers, which has an existence, independent of experience or of the human mind. One practical consequence of such a belief, already adopted by the National Aeronautics and Space Administration (Nasa), is that mathematics provides the best form of potential communication with any extraterrestrial intelligence.

Any mathematician must sympathise with Connes. We all feel that the integers, or circles, really exist in some abstract sense and the Platonist view is extremely seductive. But can we really defend it? Had the universe been one-dimensional or even discrete it is difficult to see how geometry could have evolved. It might seem that with the integers we are on firmer ground, and that counting is really a primordial notion.

But let us imagine that intelligence had resided, not in mankind, but in some vast solitary and isolated jellyfish, deep in the depths of the Pacific. It would have no experience of individual objects, only with the surrounding water. Motion, temperature and pressure would provide its basic sensory data. In such a pure continuum the discrete would not arise and there would be nothing to count.

Even more fundamentally, in a purely static universe without the notion of time, causality would disappear and with it that of logical implication and of mathematical proof. Connes actually alludes to this philosophical dilemma in the context of relativistic cosmology.

It may be argued that such "gedanken universes" are not to be taken seriously. Our actual universe is a given datum and the inevitable background of all intelligent discussion. But this is tantamount to conceding that mathematics has evolved from the human experience. Man has created mathematics by idealising or abstracting elements of the physical world. The number 2 for example represents the common attribute of all pairs of objects that we have encountered, in the same way as the word "chair" represents what is common to all the individual items of furniture that we sit on. Admittedly chairs are not so precisely and unambiguously defined as numbers: is a three-legged stool a chair? Language is inherently more fuzzy or open-ended than mathematics, but mathematics can properly be viewed as a special kind of language. This applies not only to the nouns or objects but also to the verbs or processes such as addition and subtraction.

Connes will protest that there are thousands of different languages reflecting the history and culture of a particular people, whereas mathematics is universal and unique. Surely this gives it a special status?

This can be debated at length, but a short answer is that the diversity of languages disguises a fundamental structural similarity, which is why dictionaries help us to translate. Moreover, different mathematical notations, such as the Roman numeral system, do exist but western civilization has produced a uniformity which was not inevitable.

In describing mathematics as a language it is important to emphasize that a language is not merely a set of words and grammatical rules for producing coherent sentences. Words mean something and they relate to our experience. In a similar way a mathematical statement has a meaning and one which, I would argue, rests ultimately on our experience.

For a Platonist like Connes mathematics lives in some ideal world. I find this a difficult notion to grasp and prefer to say, more pragmatically, that mathematics lives in the collective consciousness of mankind. I am not here embracing any obscure metaphysical notion, but merely observing that there are two essential components of mathematics. In the first place it deals with concepts and abstract processes which live in the conscious mind of the individual mathematician. Second, it must be communicable to other mathematicians. The famous Indian genius Ramanujan frequently produced marvellous formulae by some unknown mental process which could neither be described nor repeated. I think Connes would accept that such formulae do not become an accepted part of mathematics until others have understood them and verified their validity.

Our collective consciousness may not be sufficiently ideal for Connes but it is the world where all ideas live, not just mathematical ones.

Where does this point of view leave the dichotomy between creation and discovery in mathematics? By resisting the embrace of the Platonic world have we lost the possibility of making "discoveries"? Is every theorem man-made? Not at all.

Since our concepts are based on the physical world we can discover facts about these concepts experimentally. For example the formula 3 x 5 = 5 x 3 can be discovered by setting out 15 objects in a rectangular array. One can then move on to the more general algebraic formula a x b = b x a where a and b represent arbitrary integers. In a similar way the famous formula of Pythagoras, a2 + b2 = c2, relating the length of the sides of any right-angle triangle was no doubt discovered experimentally. So man creates the concepts of mathematics but he discovers the subsequent connections between them. The reason he can have it both ways is that mathematics is firmly rooted in the real world.

The legally minded reader may object that I have slurred the difference between an empirical discovery, always concerned with the specific, and a proof, concerned with the general. I plead guilty, but only on a technicality and I hope the judge will be lenient. Sometimes a discovery will carry with it the seeds of a formal proof, as with the example that 3 x 5 = 5 x 3. Sometimes one has to work harder as Pythagoras had to do. But if possession is nine- tenths of the law, discovery is nine-tenths of the proof.

In his dialogue with Connes, Changeux keeps hitting the Platonist rock. As a hard-headed experimental scientist Changeux wants to identify mathematics with what actually goes on in the brain. For him this is the only reality and the only place where mathematics exists. Connes disputes this extreme attitude and prefers to say that mathematical reality (which exists elsewhere) is reflected in the neurological processes of the brain. To confuse the two is like identifying a piece of literature or music with the ink and paper on which it is recorded. It is hard to disagree, but fascinating questions remain.

The brain is frequently compared to an electronic computer and artificial intelligence is now a branch of computer science. But, as the dialogue brings out, this analogy has serious limitations and may in fact be obscuring some of the essential features. As Connes notes, the speed of transmission of signals in the brain is far slower than the corresponding speeds in modern computers. As a result computers are much better and faster than humans at certain kinds of calculation, but in most important respects they are still a long way behind. For example, computers are not yet serious contributors to mathematical theory. Perhaps by analysing the structure of mathematics we may learn something about the way the brain operates. In particular, a key feature of mathematics is its hierarchical nature. Examples of patients with specific brain damage show that particular levels of reasoning are associated with definite parts of the brain.

For Changeux, the comparison between the structure of mathematics and the structure of the brain can be looked at both from the evolutionary aspect and in terms of function. In evolutionary terms, the brain has had to create a hierarchy of levels that adequately reflect the physical environment and the challenges it poses.

Man has been the ultimate winner of the evolutionary process and his brain has the structure needed to produce mathematics. Would a different neurological solution have led to a different kind of mathematics, or does mathematics depend only on the functional capacity of the brain, not on its biological mechanism?

If one views the brain in its evolutionary context then the mysterious success of mathematics in the physical sciences is at least partially explained. The brain evolved in order to deal with the physical world, so it should not be too surprising that it has developed a language, mathematics, that is well suited for the purpose. The sceptic can point out that the struggle for survival only requires us to cope with physical phenomena at the human scale, yet mathematical theory appears to deal successfully with all scales from the atomic to the galactic. Perhaps the explan-ation lies in the abstract hierarch-ical nature of mathematics which enables us to move up and down the world scale with comparative ease.

The internal workings of the mathematical mind were well described by Jacques Hadamard who distinguished three stages in attacking a problem: preparation, incubation and illumination. What these correspond to neurologically is a challenging question which generates a lively discussion between Connes and Changeux. Is there, for example, a Darwinian element in the search for successful ideas? Henri Poincare argued that the subconscious mind generates random ideas and "illumination" occurs when one of these is selected.

The original Socratic dialogues were artificially constructed to present a coherent view. The dialogue between Connes and Changeux is quite different. It is the recording of real-life arguments where the speakers are frequently at cross-purposes and operate in different planes. For the reader this can be irritating but it also encourages him to become involved and frame his own answers, as I have endeavoured to do!

Sir Michael Atiyah, OM, is president, the Royal Society, and master, Trinity College, Cambridge.

INQUIRY: AN OCCASIONAL COLUMN

# Describing Nature With Math

## How do scientists use mathematics to define reality? And why?

* By Peter Tyson
* Posted 11.10.11
* NOVA

How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of reality?
—Albert Einstein

If you're like me, you understand readily how one can describe nature's wonders using poetry or music, painting or photography. Wordsworth's "I Wandered Lonely as a Cloud" and Vivaldi's "Four Seasons" richly depict their natural subjects, as do Monet's water lilies and Ansel Adams' photos of Yosemite. But mathematics? How can you describe a tree or cloud, a rippled pond or swirling galaxy using numbers and equations?



This photograph does a pretty good job of "describing" ripples. But a mathematician could do it with greater precision and predictive power. [Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-01-large.jpg)Photo credit: © Alex Potemkin/iStockphoto

Extremely well, as Einstein knew better than most, of course. In fact, most scientists would agree that, when it comes to teasing out the inherent secrets of the universe, nothing visual, verbal, or aural comes close to matching the accuracy and economy, the power and elegance, and the inescapable truth of the mathematical.

How is this so? Well, for the math-challenged, for that person who has avoided anything but the most basic arithmetic since high school, who feels a pit in his stomach when he sees an equation—that is, for myself—I will attempt to explain, with the help of some who do mathematics for a living. If you're math-phobic, too, I think you'll get a painless feel for why even that master of describing nature with words, Thoreau, would hold that "the most distinct and beautiful statements of any truth must take at last the mathematical form."

### ANCIENT MATH

While many early civilizations, including Islamic, Indian, and Chinese, made important contributions to mathematics, it was the ancient Greeks who invented much of the math we're familiar with. Euclid fathered the geometry we named after him—all those radii and hypotenuses and parallel lines. Archimedes approximated pi. Ptolemy created a precise mathematical model that had all of the heavens wheeling around the Earth.

***"With a few symbols on a page, you can describe a wealth of physical phenomena."***

The Greeks' discoveries are timeless: Euclid's axioms are as unimpeachable today as when he devised them over 2,000 years ago. And some Greek proto-physicists did use their newfound skills to tackle mysteries of the natural world. With basic trigonometry, for example, the astronomer Eratosthenes estimated the diameter of the Earth with over 99 percent accuracy—in 228 B.C.

But while the Greeks believed that the universe was mathematically designed, they largely applied math only to static objects—measuring angles, calculating volumes of solid objects, and the like—as well as to philosophical purposes. Plato wouldn't let anyone through the front door of his acclaimed Academy who didn't know mathematics. "He is unworthy of the name of man," Plato sniffed, "who is ignorant of the fact that the diagonal of a square is incommensurable with its side." And so it remained for a millennium and a half.



Galileo strived to explain how objects fall rather than why, a *modus operandi* that set the stage for the advancement of science as we know it today.[Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-02-large.jpg)Photo credit: © Pgiam/iStockphoto

### THE MEASURE OF ALL THINGS

Galileo changed all that in the early 17th century. Eschewing the Greeks' attempts to explain why a pebble falls when you drop it, Galileo set out to determine how. The "great book" of the universe is written in the language of mathematics, he famously declared, and unless we understand the triangles, circles, and other geometrical figures that form its characters, he wrote, "it is humanly impossible to comprehend a single word of it [and] one wanders in vain through a dark labyrinth." (Wordsworth or Monet might take issue with that statement, but wait.)

Galileo sought characteristics of our world that he could measure—variable aspects like force and weight, time and space, velocity and acceleration. With such measurements, Galileo was able to construct those gems of scientific shorthand—mathematical formulas—which defined phenomena more concisely and more powerfully than had ever been possible before. (His contemporary, the German mathematician Johannes Kepler, did the same for the heavens, crafting mathematical laws that accurately describe the orbits of planets around the sun—and led to the scrapping of Ptolemy's Earth-centric model.)

### A TIDY SUM

A classic example is the formula commonly shown as d = 16t2. (Hang in there, math-phobes. Your queasiness, which I share, should go away when you see how straightforward this is.) What Galileo discovered and ensconced in this simple equation, one of the most consequential in scientific history, is that, when air resistance is left out, the distance in feet, d, that an object falls is equal to 16 times the square of the time in seconds, t. Thus, if you drop a pebble off a cliff, in one second it will fall 16 feet, in two seconds 64 feet, in three seconds 144 feet, and so on.

Galileo's succinct formula neatly expresses the notion of acceleration of objects near the surface of the Earth, but that is just the start of its usefulness. First, just as with any value of t you can calculate d, for any value of d you can figure t. To get to t, simply divide both sides of the formula d = 16t2 by 16, then take the square root of both sides. This leaves a new formula:

|  |  |
| --- | --- |
| t = âˆš | d |
| 16 |

This compact equation tells you the time needed for your pebble to fall a given distance—any distance. Say your cliff is 150 high. How long would the pebble take to reach the bottom? A quick calculation reveals just over three seconds. A thousand feet high? Just shy of eight seconds.



Boulder, pebble, pea: Despite their great differences in mass, all three objects, if dropped from our hypothetical cliff-in-a-vacuum, would reach the ground below in the same amount of time. This is what Galileo's simple formula reveals. [Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-03-large.jpg)Photo credit: © Loretta Hostettler/iStockphoto

### BROAD STROKES

What else can you do with a pithy formula like d = 16t2? Well, as hinted above, you can make calculations for an infinite number of different values for either d or t. In essence, this means that d = 16t2 contains an infinite amount of information. You can also substitute any object for your pebble—a pea, say, or a boulder—and the formula still holds up perfectly (under the conditions previously mentioned). Could a single poem or painting do as much?

***"Mathematics captures patterns that the universe finds pleasant, if you like."***

And because the same mathematical law may govern multiple phenomena, a curious scientist can discover relationships between those phenomena that might have otherwise gone undetected. Trigonometric functions, for instance, apply to all wave motions—light, sound, and radio waves as well as waves in water, waves in gas, and many other types of wave motions. The person who "gets" these trig functions and their properties will ipso facto "get" all the phenomena that these functions govern.

### A WEALTH OF DATA

The power of a potent equation extends still further. Take Isaac Newton's universal law of gravitation, which brilliantly combines Galileo's laws of falling bodies with Kepler's laws of planetary motion. Many of us know gravity vaguely as that unseen force that keeps the pebble in your palm or your feet on the ground. Newton described it this way:

|  |  |
| --- | --- |
| F = | Gm1m2 |
| r2 |

I won't go into this formula, but just know that from it you can calculate the gravitational tug between just about any two objects you can think of, from that between your coffee cup and the table it rests on, to that between one galaxy and another. Or, depending on which variables you know, you can nail down everything from the acceleration of any freely falling object near the Earth's surface (32 feet per second during every second of its fall) to the mass of our planet (about 6,000,000,000,000,000,000,000 tons).



If all other variables are known—and they are today—one can even calculate the mass of our planet using Newton's terse formula on gravitational attraction. [Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-04-large.jpg)Photo credit: Courtesy NASA

"With a few symbols on a page, you can describe a wealth of physical phenomena," says astrophysicist Brian Greene, host of [NOVA's series](http://www.pbs.org/wgbh/nova/physics/fabric-of-cosmos.html) based on his book The Fabric of the Cosmos. "And that is, in some sense, what we mean by elegance—that the messy, complex world around us emanates from this very simple equation that you have written on a piece of paper."

And like Galileo's d = 16t2, Newton's formula is amazingly accurate. In 1997, University of Washington researchers determined that Newton's inverse-square law holds down to a distance of 56,000ths of a millimeter. It may hold further, but that's as precise as researchers have gotten at the moment.

### EXACT SCIENCE

What amazes me most about Galileo and Newton's formulas is their exactitude. In Galileo's, the distance equals exactly the square of the time multiplied by 16; in Newton's, the force of attraction between any two objects is exactly the square of the distance between them. (That's the r2 in his equation.) Such exactness crops up regularly in mathematical descriptions of reality. Einstein found, for instance, that the energy bound up in, say, a pebble equals the pebble's mass times the square of the speed of light, or E = mc2.

Even things we can see and touch in nature flirt with mathematical proportions and patterns. Consider the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144â¦ Notice a pattern? After the first, every number is the sum of the previous two. The Fibonacci sequence has many interesting properties. One is that fractions formed by successive Fibonacci numbers—e.g., 3/2 and 5/3 and 8/5—get closer and closer to a particular value, which mathematicians know as the golden number. But what about this: Many plants adhere to Fibonacci numbers. The black-eyed susan has 13 petals. Asters have 21. Many daisies have 34, 55, or 89 petals, while sunflowers usually have 55, 89, or 144.



Why do sunflowers often have precisely 55, 89, or 144 petals, numbers that figure in the famous Fibonacci sequence? Nature, it seems, has certain mathematical underpinnings.[Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-05-large.jpg)Photo credit: © sefaoncul/iStockphoto

### IS GOD A MATHEMATICIAN?

The apparent mathematical nature of Nature, from forces to flowers, has left many since the time of the Greeks wondering, as the mathematician Mario Livio does in his book of the same title, "Is God a mathematician?" Does the universe, that is, have an underlying mathematical structure? Many believe it does. "Just as music is auditory patterns that the human mind finds pleasant," says Stanford mathematician Keith Devlin, "mathematics captures patterns that the universe finds pleasant, if you like—patterns that are implicit in the way the universe works."

***"Einstein used mathematics to see a piece of the universe that no one had ever seen before."***

So did we humans invent mathematics, or was it already out there, limning the cosmos, awaiting the likes of Euclid to reveal it? In his book Mathematics in Western Culture, the mathematician Morris Kline chose to sidestep the philosophical and focus on the scientific: "The plan that mathematics either imposes on nature or reveals in nature replaces disorder with harmonious order. This is the essential contribution of Ptolemy, Copernicus, Newton, and Einstein."

### SEEING THE INVISIBLE

Formulas like Galileo's and Netwon's make the invisible visible. With d = 16t2, we can "see" the motion of falling objects. With Newton's equation on gravity, we can "see" the force that holds the moon in orbit around the Earth. With Einstein's equations, we can "see" atoms. "Einstein is famous for a lot of things, but one thing that is often overlooked is he's the first person who actually said how big an atom is," says Jim Gates, a physicist at the University of Maryland. "Einstein used mathematics to see a piece of the universe that no one had ever seen before."

Today, with advanced technology, we can observe individual atoms, but some natural phenomena defy any description but a mathematical one. "The only thing you can say about the reality of an electron is to cite its mathematical properties," noted the late mathematics writer Martin Gardner. "So there's a sense in which matter has completely dissolved and what is left is just a mathematical structure." Charles Darwin, who admitted to having found mathematics "repugnant" as a student, may have put it best when he wrote, "Mathematics seems to endow one with something like a new sense."



Mathematics predicted what nature has long known—that the stripes on the marine angelfish actually migrate across its body over time.[Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-06-large.jpg)Photo credit: © Iliuta Goean/iStockphoto

### FORTUNE TELLING

Mathematics also endows one with an ability to predict, as Galileo's and Newton's formulas make clear. Such predictive capability often leads to new discoveries. In the mid-1990s, Kyoto University researchers realized to their surprise that equations originally devised by the mathematical genius Alan Turing predicted that the parallel yellow and purple stripes of the marine angelfish have to move over time. Stable, unmoving patterns didn't jive with the mathematics. To find out if this was true, the researchers photographed angelfish in an aquarium over several months. Sure enough, an angelfish's stripes do migrate across its body over time, and in just the way the equations had indicated. Math had revealed the secret.

"There really is a facing-the-music that math forces, and that's why it's a wonderful language for describing nature," Greene says. "It does make predictions for what should happen, and, when the math is accurately describing reality, those predictions are borne out by observation."

### A MATH FOR ALL SEASONS

So much mathematics exists now—one scholar estimates that a million pages of new mathematical ideas are published each year—that when scientists face problems not solvable with math they know, they can often turn to another variety for help. When Einstein began work on his theory of general relativity, he needed a mathematics that could describe what he was proposing—that space is curved. He found it in the non-Euclidean geometry of 19th-century mathematician Georg F. B. Riemann, which provided just the tool he required: a geometry of curved spaces in any number of dimensions.



With fractal geometry, you can write down formulas that describe "rough" shapes like trees, in contrast to "smooth" shapes like ripples. [Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-07-large.jpg)Photo credit: © Chris Hepburn/iStockphoto

Or, if necessary, they invent new math. When the late mathematician Benoit Mandelbrot concluded that standard Euclidean geometry, which is all about smooth shapes, fell short when he tried to mathematically portray "rough" shapes like bushy trees or jagged coastlines, he invented a new mathematics called fractal geometry. "Math is our one and only strategy for understanding the complexity of nature," says Ralph Abraham, a mathematician at the University of California Santa Cruz, in NOVA's [Hunting the Hidden Dimension](http://www.pbs.org/wgbh/nova/physics/hunting-hidden-dimension.html). "Fractal geometry has given us a much larger vocabulary, and with a larger vocabulary we can read more of the book of nature." Galileo would be so proud.

### TECHNOLOGICAL WONDERS

Galileo would also be proud of just how much his successors have achieved with his scientific method. Formulas from his own on falling bodies to Werner Heisenberg's on quantum mechanics have provided us the means to collect and interpret the most valuable knowledge we have ever attained about the workings of nature. Altogether, the most groundbreaking advances of modern science and technology, both theoretical and practical, have come about through the kind of descriptive, quantitative knowledge-gathering that Galileo pioneered and Newton refined.

***"Do not worry about your difficulties in mathematics; I can assure you that mine are still greater."***

Newton's law of gravity, for instance, has been critical in all our missions into space. "By understanding the mathematics or force of gravity between lots of different bodies, you get complete control and understanding, with very high precision, of exactly the best way to send a space probe to Mars or Jupiter or to put satellites in orbit—all of those things," says Ian Stewart, an emeritus professor of mathematics at the University of Warwick in England. "Without the math, you would not be able to do it. We can't send a thousand satellites up and hope one of them gets into the right place."



Without Newton's formula on gravitational attraction, we would never have been able to send satellites and other craft into space so successfully. Here, the International Space Station as seen in 2007.[Enlarge](http://www.pbs.org/wgbh/nova/assets/img/describing-nature-math/image-08-large.jpg)Photo credit: Courtesy NASA

Mathematics underlies virtually all of our technology today. James Maxwell's four equations summarizing electromagnetism led directly to radio and all other forms of telecommunication. E = mc2 led directly to nuclear power and nuclear weapons. The equations of quantum mechanics made possible everything from transistors and semiconductors to electron microscopy and magnetic resonance imaging.

Indeed, many of the technologies you and I enjoy every day simply would not work without mathematics. When you do a Google search, you're relying on 19th-century algebra, on which the search engine's algorithms are based. When you watch a movie, you may well be seeing mountains and other natural features that, while appearing as real as rock, arise entirely from mathematical models. When you play your iPod, you're hearing a mathematical recreation of music that is stored digitally; your cell phone does the same in real time.

"When you listen to a mobile phone, you're not actually hearing the voice of the person speaking," Devlin told me. "You're hearing a mathematical recreation of that voice. That voice is reduced to mathematics."

### AFTERMATH

And I'm reduced to conceding that math doesn't scare me so much anymore. How about you? If you still feel queasy, perhaps you can take solace from Einstein himself, who once reassured a junior high school student, "Do not worry about your difficulties in mathematics; I can assure you that mine are still greater."

Peter Tyson is editor in chief of NOVA Online.

<http://www.pbs.org/wgbh/nova/physics/describing-nature-math.html>

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**The Structure of Everything**
Did we invent math? Or did we discover it?

Reviewed by Marc Kaufman
Sunday, February 8, 2009

*IS GOD A MATHEMATICIAN?*

*By Mario Livio*

*Simon & Schuster. 308 pp. $26*

Did you know that 365 -- the number of days in a year -- is equal to 10 times 10, plus 11 times 11, plus 12 times 12?

Or that the sum of any successive odd numbers always equals a square number -- as in 1 + 3 = 4 (2 squared), while 1 + 3 + 5 = 9 (3 squared), and 1 + 3 + 5 + 7 = 16 (4 squared)?

Those are just the start of a remarkable number of magical patterns, coincidences and constants in mathematics. No wonder philosophers and mathematicians have been arguing for centuries over whether math is a system that humans invented or a cosmic -- possibly divine -- order that we simply discovered. That's the fundamental question Mario Livio probes in his engrossing book *Is God a Mathematician?*

Livio, an astrophysicist at the Hubble Space Telescope Science Institute in Baltimore, explains the invention-vs.-discovery debate largely through the work and personalities of great figures in math history, from Pythagoras and Plato to Isaac Newton, Bertrand Russell and Albert Einstein. At times, Livio's theorems, proofs and conundrums may be challenging for readers who struggled through algebra, but he makes most of this material not only comprehensible but downright intriguing. Often, he gives a relatively complex explanation of a mathematical problem or insight, then follows it with a "simply put" distillation.

An extended section on knot theory is, well, pretty knotty. But it ultimately sheds light on the workings of the DNA double helix, and Livio illustrates the theory with a concrete example: Two teams taking different approaches to the notoriously difficult problem of how many knots could be formed with a specific number of crossings -- in this case, 16 or fewer -- came up with the same answer: 1,701,936.

The author's enthusiasm is infectious. But it also leads him to refer again and again to his subjects as "famous" and "great" and to their work as "monumental" and "miraculous." He has a weakness as well for extended quotes from these men (and they are *all* men) that slow the narrative without adding much. There are exceptions, including the tale of how Albert Einstein and mathematician Oskar Morgenstern tried to guide Kurt Gödel, a fellow mathematician and exile from Nazi Germany, through the U.S. immigration process.

A deep-thinking and intense man, Gödel threw himself into preparing for his citizenship test, including an extremely close reading of the U.S. Constitution. In his rigorously logical analysis, he found constitutional weaknesses that he thought could allow for the rise of a fascist dictatorship in America. His colleagues told him to keep that reading to himself, but he blurted it out during his naturalization exam. He was allowed to stay anyway.

The interplay of mini-biography and the march of mathematical knowledge serves the author well. It does not, however, ultimately help him to answer the big question, Is God a mathematician?

On one side of the debate are all those remarkable constants that crop up, the makings of the ideal yet hidden world posited by Plato. In addition, there's what the physicist Eugene Wigner, in a seminal 1960 essay, called the ["unreasonable effectiveness"](http://www.dartmouth.edu/%7Btilde%7Dmatc/MathDrama/reading/Wigner.html) of mathematical theorems: the astounding ability of math to predict unimagined results. Wigner was picking up on ideas explored earlier by Einstein, and Einstein's general theory of relativity remains one of the best examples: His predictions about how gravity can cause ripples in space-time was recently corroborated by measuring radio waves from a distant set of compact, high-energy stars called double pulsars, using technology unknown in Einstein's day. Doesn't all this indicate that the mathematical structure of the world is out there waiting to be discovered?

On the other hand, math cannot explain many situations, and chaos theory suggests that it may never be possible to predict the weather or the stock market with accuracy. Recent research has pointed to basic mathematical constructs in the human brain, suggesting that we impose numbers and forms on the world, not vice versa. In addition, mathematics is less stable than it appears to us in grade school. At the higher reaches of the field, there is constant ferment and debate. If the "truths" discovered through mathematics are always changing, doesn't that indicate they are a product of human study and manipulation, rather than something fixed and eternal?

As explained by Livio, the history of mathematics is partly a struggle between these points of view: that math is how God (or nature) organizes the world, or it is simply a human tool to understand that world.

Livio comes down in the middle, contending that math may well be both invented and discovered. He points, for instance, to the eternal truth contained in the geometry formulated by Euclid 2,400 years ago. By the 19th century, however, iconoclasts had posited and established a whole new world of non-Euclidian geometry. Livio writes about the symmetries of the universe: the immutable, if incompletely understood, laws of math and physics that make a hydrogen atom, for instance, behave in the same way on Earth as it acts 10 billion light years away. Another sign of universal structure, as teased apart with the help of math? No, Livio writes, it is more likely a sign that "to some extent, scientists have selected what problems to work on based on those problems being amenable to a mathematical treatment."

The author acknowledges that some readers will find his inconclusive conclusion to be unsatisfying. I didn't. Sometimes the adventure, the intellectual ride, is more important than the final destination. ·

*Marc Kaufman, a science reporter for The Washington Post, is writing a book about astrobiology.*

<http://www.washingtonpost.com/wp-dyn/content/article/2009/02/05/AR2009020502876_pf.html>

# How Math Works

**BY**[**ROBERT LAMB**](http://www.howstuffworks.com/robert-lamb-author.htm)[**SCIENCE**](http://science.howstuffworks.com/)**|**[**MATH CONCEPTS**](http://science.howstuffworks.com/math-concepts)

Browse the article [How Math Works](http://science.howstuffworks.com/math-concepts/math.htm)



**Don't fear the math.**

**PETER CADE/ICONICA/**[**GETTY IMAGES**](http://www.gettyimages.com/)

It's easy to think of mathematics as a kind of storybook sorcery -- a powerful secret language known to few, mastered by inhuman agents (such as your calculator) and underpinning the very fabric of the universe. Even if we avoid such hyperbole, the fact remains: Many of us are mathematically illiterate in a world that runs on math.

When was the last time you seriously crunched some numbers with only pen and paper? In his book "The Geometry of Paradise," Mark A. Peterson described the people of [medieval](http://history.howstuffworks.com/10-medieval-torture-devices.htm) Europe as a nonmathematical culture in possession of sophisticated mathematics. Mathematicians of the day certainly honed their skills but mostly out of love for mathematical abstractions. They perused few practical applications with it and, according to Peterson, didn't really grasp what math was.

Today, the mathematics field is far more vibrant than it was in the Middle Ages, but it still eludes an alarming number of those who depend on it. On one hand, math certainly has a way of solving itself these days through calculators and hastily keyed-in Google searches. Yet for many individuals, mathematical anxiety begins with inadequate teaching from nonmathematicians who have trouble relaying enthusiasm and practicality. Factor in overcrowded classes, and it's little wonder that so many students fail to latch onto math's logical core. In fact, only 40 percent of 4th graders and 34 percent of 8th graders in the U.S. are proficient in math, according to Arne Duncan, U.S. [education](http://people.howstuffworks.com/public-schools.htm) secretary speaking at the National Council of Teachers of Mathematics in April 2011.

The ramifications of mathematical illiteracy are very real. In 2005, the United States National Academies identified the country's decline in mathematics education as having a severe detrimental effect on its scientific, technological and economic prowess [source: [Mullich](http://online.wsj.com/ad/article/mathscience-rising)].

So let's demystify the world of mathematics. A world without math is unimaginable. It's a part of who we are. It's the analytical juice of our left [brain](http://health.howstuffworks.com/human-body/systems/nervous-system/brain.htm) and, in the words of physicist Richard Feynman, even a fool can use it. Here's a quote from the late great scientist's book "The Pleasure of Finding Things Out":

What we've been able to work out about nature may look abstract and threatening to someone who hasn't studied it, but it was fools who did it, and in the next generation, all the fools will understand it. There's a tendency to pomposity in all this, to make it deep and profound.

In this article, we'll take a very wide-angle look at the world of numbers. Just what are they, and what does math really do?

## What Are Numbers?



**A boxing referee administers the count.**

**PICTORIAL PARADE/**[**GETTY IMAGES**](http://www.gettyimages.com/)

Mathematics boils down to pattern recognition. We identify [patterns](http://science.howstuffworks.com/math-concepts/tessellations.htm) in the world around us and use them to navigate its challenges. To do all this, however, we need numbers -- or at least the [information that our numbers represent](https://curiosity.com/playlists/a-new-way-to-think-about-numbers-curiosity).

What are numbers? As we'll explore more later, that's a deceptively deep question, but you already know the simple answer. A number is a word and a symbol representing a count. Let's say you walk outside your home and you see two angry dogs. Even if you didn't know the word "two" or know what the corresponding numeral looks like, your brain would have a good grasp of how a two-dog encounter compares with a three-, one- or zero-dog situation.

We owe that innate comprehension to our [brain](http://health.howstuffworks.com/human-body/systems/nervous-system/brain.htm) (specifically, the inferior parietal lobe), which naturally extracts numbers from the surrounding environment in much the same way it identifies colors [source: [Dehaene](http://www.edge.org/3rd_culture/dehaene/dehaene_p2.html)]. We call this **number sense**, and our brains come fully equipped with it from birth. Studies show that while infants have no grasp of human number systems, they can still identify changes in quantity.

Neuroimaging research has even discovered that [infants](http://lifestyle.howstuffworks.com/family/parenting/babies/understanding-cognitive-and-social-development-in-a-newborn-ga.htm) possess the ability to engage in **logarithmic counting**, or counting based on integral increases in physical quantity. While a baby won't see the difference between five teddy bears and six teddy bears in a lineup, he or she will notice a difference between five and 10 [source: [Miller](http://www.radiolab.org/2009/nov/30/innate-numbers/)].

Number sense plays a vital role in the way animals navigate their environments -- environments where objects are numerous and frequently mobile. However, an animal's numerical sense becomes more imprecise with increasingly larger numbers. Humans, for instance, are systematically slower to compute 4 + 5 than 2 + 3 [source: [Dehaene](http://www.edge.org/3rd_culture/dehaene/dehaene_p2.html)].

At some point in our ancient past, prehistoric humans began to develop a means of augmenting their number sense. They started counting on their fingers and toes. This is why so many numerical systems depend on groups of five, 10 or 20. Base-10 or **decimal systems** stem from the use of both hands, while base-20 or **vigesimal systems** are based on the use of fingers and toes.

So ancient humans learned to externalize their number sense and, in doing so, they arguably created humanity's most important [scientific achievement](http://science.howstuffworks.com/innovation/scientific-experiments/5-scientific-breakthroughs-we-couldnt-live-without.htm): mathematics.

## The Tower of Math: Numbers

Numbers pose a difficulty for humans. Sure, some of us have more of a gift for math than others, but every one of us reaches a point in our mathematical education where things become hard. Learning your multiplication tables is difficult because the human brain never evolved to handle such advanced computations as 17 x 32 = 544. After a certain point, our mathematical education is largely an exercise in rejiggering ill-adapted [brain](http://health.howstuffworks.com/human-body/systems/nervous-system/brain.htm) circuits [source: [Dehaene](http://www.edge.org/3rd_culture/dehaene/dehaene_p2.html)].

Number sense may come naturally to us, but mathematical literacy comes only with time. Likewise, humanity's use of mathematics has steadily grown over the ages. Like science itself, math isn't the product of one mind but rather a steady accumulation of knowledge throughout human history.

Think of math as a tower. Natural human height is finite, so if we're to reach higher into the air and see out farther across the landscape, we'll need to build something external to ourselves. Our mental abilities to understand math are equally finite, so we build a great tower of number systems and climb upward to the [stars](http://science.howstuffworks.com/star.htm).

To break down the basic structure of this tower, let's first look at the raw materials. These are the basic types of numbers:

**Integers:** You probably know these as whole numbers, and they come in both positive and negative forms. Integers include the basic counting numbers (1-9), negative numbers (-1) and zero.

**Rational numbers** include integers but also encompass simple fractions that can be expressed as a ratio of two integers. For example, 0.5 is rational because we can also write it as 1/2.

**Irrational numbers**: These numbers can't be written as a ratio of two integers. Pi (the ratio of the circumference of a circle to its diameter) is a classic example, as it can't be written accurately as a ratio of two integers and has been calculated to trail off decimal points into the trillions.

Rational and irrational numbers both fall under the category of **real numbers** or **complex numbers**. And yes, there are also **imaginary numbers** that exist outside the real number line, and **transcendental numbers**, such as [pi](http://science.howstuffworks.com/math-concepts/pi.htm). There are many other different numbers types as well, and they, too, play a part in the structure of our tower.

On the next page, we'll look at some of the core branches of mathematics.

## The Tower of Math: Branches of Mathematics



**Circa 100 B.C., Greek astronomer Hipparchus, inventor of trigonometry, studies the heavens.**

**ARCHIVE PHOTOS/**[**GETTY IMAGES**](http://www.gettyimages.com/)

Who would you hire to build a tower? After all, several different systems converge in [modern construction](http://home.howstuffworks.com/home-improvement/construction/materials/5-long-lasting-building-materials.htm): steel framework, stone foundation, woodwork, plumbing, roofing, electrical wiring, telecommunications heating and air conditioning. Likewise, many branches of mathematics play a part in the tower of math. Here are just a few.

**Arithmetic**: This is the oldest and most basic form of mathematics. Arithmetic chiefly concerns the addition, subtraction, multiplication and division of real numbers that aren't negative.

**Algebra**: The next level of mathematics, algebra, is essentially arithmetic with unknown or abstract quantities thrown in with the real numbers. We represent the abstracts with symbols, such as X and Y.

**Geometry**: Remember what we said about math helping us navigate a world of numerous and movable objects? This is where geometry comes into play, dealing chiefly with the measurements and properties of points, lines, angles, surfaces and solids.

**Trigonometry**: Trigonometry concerns the measurements of triangles and the relationships between their sides and angles. While the historical origins of arithmetic, algebra and geometry are lost in the fog of ancient history, trigonometry originates with second century astronomer Hipparchus of Nicaea.

**Calculus**: Independently developed by both [Isaac Newton](http://science.howstuffworks.com/dictionary/famous-scientists/physicists/isaac-newton.htm) and Gottfried Leibniz in the 17th century, calculus deals with the calculation of instantaneous rates of change (known as **differential calculus**) and the summation of infinite small factors to determine some whole (known as **integral calculus**). As such, it has proven a vital scientific tool in a number of disciplines.

The tower of mathematics has enabled human culture to rise and flourish, to understand both the inner mysteries of the cells to the outer mysteries of space.

But did we truly build this tower out of our own ingenuity? Did we invent mathematics or merely discover it? Explore this tantalizing question on the next page.

## Math: Human Discovery or Human Invention?



**Does the universe conform to math, or math to the universe?**

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So just what, in essence, is this thing called math? In developing these numbers and systems of numbers, did we discover the hidden coding of the universe? Is mathematics, in the words of [Galileo](http://science.howstuffworks.com/innovation/famous-inventors/were-galileos-inventions-all-about-the-cosmos.htm), the language of God? Or is math just a human-created system that happens to correspond with natural laws and structures? There is no definitive answer to this question, but mathematicians tend to side with one of several compelling theories.

First, there is the **Platonic theory**. Greek philosopher Plato argued that math is a discoverable system that underlines the structure of the universe. In other words, the universe is made of math and the more we understand this vast interplay of numbers, the more we can understand nature itself. To put it more bluntly, mathematics exists independent of humans -- that it was here before we evolved and will continue on long after we're extinct.

The opposing argument, therefore, is that math is a man-made tool -- an abstraction free of [time and space](http://science.howstuffworks.com/science-vs-myth/everyday-myths/see-the-fourth-dimension.htm) that merely corresponds with the universe. Just consider elliptical planetary orbits. While such an elliptical trajectory provides astronomers with a close approximation of the planet's movement, it's not a perfect one [source: [Dehaene](http://www.edge.org/3rd_culture/dehaene/dehaene_p2.html)].

Several theories expand on this idea.

* The **logistic theory**, for instance, holds that math is an extension of human reasoning and logic.
* The **intuitionist theory** defines math as a system of purely mental constructs that are internally consistent.
* The **formalist theory** argues that mathematics boils down to the manipulation of man-made symbols. In other words, these theories propose that math is a kind of analogy that draws a line between concepts and real events.
* The **fictionalist theory**, while less popular, even goes so far as to equate mathematics with fairy tales: scientifically useful fictions. In other words, 1 + 1 = 2 might enable us to understand how the universe works, but it isn't a "true" statement.

Who's right? Who's wrong? There's ultimately no way to know, but on the next page we'll look at two examples of what each possibility could mean to our understanding of the universe.

## The Mathematical Universe



**Can math explain it all?**

**IMAGE COURTESY**[**NASA**](http://www.nasa.gov/)**/JPL-CALTECH/R. KENNICUTT/SINGS**

The history of mathematics is a history of humanity seeking to understand the universe. Therefore, many consider the holy grail of mathematics to be the same as that of physics: a [**theory of everything**](http://science.howstuffworks.com/science-vs-myth/everyday-myths/theory-of-everything.htm), a unified theory that explains all physical reality.

Math generally plays a vital role in any theory of everything, but contemporary cosmologist Max Tegmark even goes so far as to theorize that the universe itself is made of math. In his **mathematical universe hypothesis**, he proposes that math is indeed a human discovery and that the universe is essentially one gigantic mathematical object. In other words, mathematics no more describes the universe than atoms describe the objects they compose; rather math is the universe. Tegmark even goes so far as to predict that a mathematical proof for a theory of everything could eventually fit on a T-shirt.

More than 60 years earlier, however, Austrian mathematician Kurt Gödel put forth a theory that argues quite the opposite. **Gödel's first incompleteness theorem** concerns axioms, logical mathematical statements that we assume to be true but can't be proven with a mathematical proof. A simple example of this would be the axiom of equality (X = X). We assume this to be a true statement, but we can't actually back it up with a mathematical proof. Gödel's theorem states that any adequate axiomatizable theory is incomplete or inconsistent.

The implication, according to theoretical physicist and mathematician Freeman Dyson, is that mathematics is inexhaustible. No matter how many problems we solve, we'll inevitably encounter more unsolvable problems within the existing rules [source: [Feferman](http://math.stanford.edu/~feferman/papers/Godel-IAS.pdf)]. This would also seem to rule out the potential for a theory of everything, but it still doesn't relegate the world of numbers to either human invention or human discovery.

Regardless, mathematics could stand as humanity's greatest invention. It composes a vital part of our [neural architecture](http://health.howstuffworks.com/human-body/systems/nervous-system/brain.htm) and continues to empower us beyond the mental limits we were born with, even as we struggle to fathom its limits.

Explore the links on the next page to learn even more about mathematics.

### Related Articles

* [How are Fibonacci numbers expressed in nature?](http://science.howstuffworks.com/math-concepts/fibonacci-nature.htm)
* [How Tessellations Work](http://science.howstuffworks.com/math-concepts/tessellations.htm)
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